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Calculation of the masses and the running masses of the quarks and leptons from electroweak to supersymmetric grand unification

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Abstract We make a systematic theoretical analysis of the masses of the fermions and their variation with energy by solving the one loop renormalization group equation (RGE) in the Minimal Supersymmetric Standard Model (MSSM) A simple common mass for all the fermions around 115 GeV at GUT scale has been found as a possible solution of RGE by Deo Maharana and Mishra Here we undertake the unfinished but the important task of calculating the electroweak masses of the fermions at different energies The proposed parametric unification mass and group theoretic constants for the model are well known The mass of the top quark and its descent is studied by an approximate method very carefully We find that the Ramond Roberts and Ross value of the Wolfenstein parameter is reproduced and is nearly equal to 0 22 When raised to integral powers and multiplied by 115 GeV the whole mass spectra of the remaining eleven fermions are obtained within experimental errors We deduce formulae for the masses and plot them for all the 12 fermions from $t - \log(\mu/M_x) = 0$ to $t_x = 33$ the GUT mass being $M_x = 2\ 2 \times 10^{16}$ GeV

Keywords RGE MSSM

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1. Introduction

Pati and Salam [1] pioneered the idea that leptons are the fourth colour, quarks and leptons should be brought under the same umbrella of one group so that all forces (except gravity) can be understood in terms of one unifying force parameter near the Planck scale The six leptons should be treated at par with the six coloured quarks The elementary constituents of matter would become twelve only With the availability of

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enormous data from the high energy accelerators, phenomenological analysis backed by imaginative theories, it has been found that the familiar standard model, which is the product group $SU_C(3)\otimes SU_L(2)\otimes Y_y(1)$, can explain all of them successfully This standard model is characterised by three coupling strengths of the weak, the electromagnetic and the strong interactions It was conceived that all the three couplings should run, *i e*, they change with energy, and eventually become one at the grand unified scale This could be made possible by solving the relevant RGE The coefficients of the theory are specified by the group structure alone Many theoretical models were investigated and only a few years back, it has become essential to bring in supersymmetry In the minimal version of the MSSM, the beta coefficients are such that they run the coupling strengths to a single value $\left(4\pi/q_U^2\right)=\alpha_{\rm GUT}\doteq 1/25$ at a mass of $M_X = 2\ 2\times 10^{16}$ GeV This has been the most attractive result in the current investigations in gauge theories [2]

Next in order, the major challenge in particle physics, was and is the theoretical derivation of the mass spectrum of the quarks and leptons in the same successful MSSM theory In this model, all the masses of the fermions and the mixing angles were being chosen arbitrarily to account for the 19 free parameters of the theory In the absence of such a fundamental theory, it has been in vogue to pursue the method which has been known as 'texture analysis' After finding a suitable texture, one can investigate further and possibly obtain unification mass parameters for all the fermions as a generalisation of the hypothesis by Georgi *et al* [3] Eventually, one can predict their individual masses by using the analytical MSSM group coefficients for a RGE for the masses of the fermions As yet, this programme has not been entirely successful, even though several attempts have been made There has been many numerical analysis using high speed computers The reason is, perhaps, not hard to seek The integrands of the mass equation can be mapped into a unit circle in complex (t-log (mass)) plane There are poles at the mass values in the integrand in different Riemann sheets To find them, by pure analytic means is very involved and may be impossible It is much easier to find the poles and the residues, though not exact, they are quite accurate for our purpose

Unfortunately, analytical solutions had not been searched for by finding the solutions of the equations by looking at the similarity of the forms of the equations of the gauge sector Moreover, the integrals to be evaluated to solve the differential equations can be multivalued We have given exact one loop solutions in eqs (2 16) and (2 17) From these we find, as already mentioned, the masses turn out to be poles in different Riemann sheets when the whole of the complex $t=\log\mu/M_Z$ plane is mapped into inside of a circle The analytical details will be discussed in the Section 4 of the paper

One of the 13 parameters of the standard model was first predicted in 1974 by Gaillard and Lee [4] Then came the popular mass matrix ansatz of Fritzsch [5] A complete listing of textures and their relevance to experimental findings were made by Ramond, Roberts and Ross (RRR) [6] Since they also took the help of RGE, we shall present the essential results of their work in this section Before that, we present the MSSM Lagrangian

$$L_{M} = \bar{Q}_L M_U \Phi_u U_R + \bar{Q}_L M_D \Phi_d D_R + \bar{l}_L M_E \Phi_d E_R + \bar{\nu}_L M_N \Phi_u \nu_R + h\,c \tag{1 1}$$

We ignore the particles and consider the rigid part of the Lagrangian given by Demir [7]

In the renormalisation schemes, the Yukawa couplings $M_F(t)$ and the v e v $v_F(t)$ change with energy differently The Dirac masses are given by

$$m_F(t) = v_F(t)\, M_F(t) \tag{1 2}$$

The masses considered above are not the masses of the flavor eigenstates of the model

The one loop RGE, as written by Grzadkowski, Lindner and Theisen [8] for $M_Y(t)$ are

$$16\pi^2 \frac{dM_Y}{dt} = (-G_Y + T_Y + S_Y)\, M_Y(t), \tag{1 3}$$

$t = \log(\mu/M_Z)$, μ is the renormalisation point and M_Z is the mass of the Z-boson Here $Y = U$ stands for U-quarks (F = 1, 2, 3), u, c and t, $Y = D$ stands for Down quarks (F = 4, 5, 6), b, s and d, $Y = N$ stands for Neutrinos (F = 7, 8, 9), ν_e, ν_μ, and ν_τ and $Y = E$ stands for electrons (F = 10, 11, 12) e μ and τ $G_Y(t)$ contains the gauge coupling terms, given in Section 2 and

$$T_U = Tr\left(3M_U M_U^\dagger + M_N M_N^\dagger\right),$$

$$T_D = Tr\left(3M_D M_D^\dagger + M_E M_E^\dagger\right), \quad T_U = T_N\,, T_E = T_D \tag{1 4}$$

and

$$S_U = 3M_U M_U^\dagger + M_D M_D^\dagger,\ S_D = 3M_D M_D^\dagger + M_U M_U^\dagger,\ S_E = 3M_E M_E^\dagger + M_N M_N^\dagger\,,$$

$$S_N = 3M_N M_N^\dagger + M_E M_E^\dagger \tag{1 5}$$

To find the couplings, we have to solve the twelve differential equations and determine all the couplings/masses from only one value of coupling at $t = t_X$ or $M_F(M_X)$ First, we turn our attention to the mas of the top quark Incidentally, Pendleton and Ross [9], Faraggi [10], and some others have predicted the value of the mass of the top quark which was around 175 GeV, even before the top was discovered

In this paper, all the one loop equations are solved following the same method used for the gauge sector A particular $M_F(M_Z)$ is obtained relating to $M_F(M_X)$ and other fermions Eventhough we aim at single input value $M_F(M_X)$ = M_U, which is independent of F, the integrals needed for the solutions for the masses are such that

$$\int M_{\rm top}^2(\tau)\, d\tau >> \int M_{Q,\rm top}^2(\tau)\, d\tau >>> \int M_{\rm lepton}^2(\tau)\, d\tau\,. \tag{1 6}$$

However, average quark masses for the integral from t to t_X, tend to be equal, the first inequality for eq (1 6) is nearly equal to one The leptonic ones are still negligible Neglect of the masses other than the top quark, gives a reliable result for the $M_{\text{top}}(M_X) \doteq M_U$ Following this approximation procedure, we find from the other eleven equations that $M_{\text{top}}(M_X) \doteq M_F(M_X) \doteq M_U \doteq 115$ GeV for the all other fermions, using the equations for heavy quarks given by RRR

We choose $M_U \doteq 115$ GeV as the only input Furthermore, MSSM has two Higgs

$$\langle \phi_U \rangle = v_u(t) = v(t)\sin\beta(t), \quad \langle \phi_D \rangle = v_d(t) = v(t)\cos\beta(t), \quad v^2(t) = v_d^2(t) + v_u^2(t)$$

$$\text{and} \qquad \tan\beta(t) = \frac{v_u(t)}{v_d(t)} \tag{1 7}$$

It is to be noted that $v(t)$ is the vacuum expectation value of single Higgs of the Standard Model, $v(M_Z) = 246$ GeV, $\sin\beta_{\text{SM}}(M_Z) = 1/\sqrt{2}$ We note that $v_0 = 246/\sqrt{2}$ GeV = 174 GeV, which is close to $M_{\text{top}}(M_Z) \cong 175$ GeV For simplicity, we shall use this value as the unit of energy whenever unspecified

The one loop RGE are

$$16\pi^2 \frac{dv_u}{dt} = \left(\frac{2}{20}g_1^2 + \frac{3}{4}g_2^2 - Tr\left(3M_U M_U^\dagger\right)\right) v_u\,,$$

$$\text{and} \qquad 16\pi^2 \frac{dv_u}{dt} = \left(\frac{2}{20}g_1^2 + \frac{3}{4}g_2^2 - Tr\left(3M_D M_D^\dagger\right)\right) v_d \tag{1 8}$$

The calculation of $\tan\beta(t) = (v_u(t))/(v_d(t))$ in MSSM has attracted considerable attention There exist extensive literature, most of them are given in reference [11] We note that for the normal SM, one Higgs v e v satisfies the one loop equation

$$16\pi^2 \frac{dv_{\text{SM}}}{dt} = \left(\frac{9}{20}g_1^2 + \frac{3}{4}g_2^2 - Tr\left(3M_U M_U^\dagger + 3M_D M_D^\dagger\right)\right) v_{\text{SM}} \tag{1 9}$$

However, for a transition from MSSM to continue below M_Z, we have to deal with the defining eq (1 7)

$$\frac{1}{2}16\pi^2 \frac{d}{dt}\left(v_u^2 + v_d^2\right) = \left(\frac{3}{20}g_1^2 + \frac{3}{4}g_2^2 - Tr\left(3M_U M_U^\dagger\right)\right) v_u^2$$

$$+\left(\frac{3}{20}g_1^2 + \frac{3}{4}g_2^2 - Tr\left(3M_D M_D^\dagger\right)\right) v_d^2, \tag{1 10}$$

As v_u tends to v_d, below M_Z for broken MSSM, we have

$$16\pi^2 \frac{dv_{\text{SM}}}{dt} = \left(\frac{3}{20}g_1^2 + \frac{3}{4}g_2^2 - Tr\left(3M_U M_U^\dagger + 3M_D M_D^\dagger\right)\right) v_{\text{SM}} \tag{1 11}$$

To maintain continuity, we assume that at $t = 0$,

$$\tan\beta(M_Z) = \tan\beta_{\text{SM}}(M_Z) = 1 \tag{1 12}$$

Using the approximation given in eq (1 6), we get from eq (1 8),

$$\tan\beta(t) \cong \exp\left(-\frac{3}{16\pi^2}\int_0^t \left[M_{\text{top}}^2(\tau) - M_{\text{bottom}}^2(\tau)\right] d\tau\right) \tag{1 13}$$

Using the fact the t integrals for the heavy quarks in Section 3 are nearly the same and close to one and deduced in some detail in Section 3 and we find that $\tan\beta(t)$ is a slowly varying function of t and drops from 1 at 91 GeV to 0 9 at 10^{16} GeV So the calculation is much simplified if we assume that $\sin\beta = 1/\sqrt{2} = \cos\beta$ This value is not ruled out by experiments [8]

Dimopoulos, Hall and Raby [12] have analysed and included the leptons following Georgi [3] and solved the MSSM RGE with some degree of success RRR tried to analyse all the cases and put them in a CKM matrix form, proposed by Wolfenstein [13] and diagonalise them to the texture types

$$V_{\text{CKM}} = \begin{pmatrix} c_1c_2 - s_1s_2e^{-i\phi} & s_1 + c_1s_2e^{-i\phi} & s_2(s_3 - s_4) \\ -c_1s_2 - s_1e^{i\phi} & -s_1s_2 + (c_1c_2c_3c_4 + s_1s_4)e^{-i\phi} & s_3 - s_4 \\ s_1(s_3 - s_4) & -c_1(s_3 - s_4) & (c_3c_4 + s_1s_4)e^{i\phi} \end{pmatrix} \tag{1 14}$$

$$= \begin{pmatrix} \lambda^2/2 & \lambda & A\lambda^3(\rho + i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho + i\eta) & -A\lambda^3 & 1 \end{pmatrix} \tag{1 15}$$

Here s_i, c_i (i = 1, , 4) are the sines and cosines of mixing angles The parameters λ can be identified as the Wolfenstein parameter From the identities given by Dimopoulos, Hall and Raby [12], following from eqs (1 14) and (1 15)

$$\lambda = \left(s_1^2 + s_2^2 + s_1s_2\cos\phi\right)^{1/2} \tag{1.16}$$

ϕ is the CKM phase angle and $s_1 = (M_d/M_s)^{1/2} = \lambda$, $s_2 = (M_u/M_c)^{1/2} = \lambda^2$, and $s_4 = ((M_d M_s)/M_b^2)^{1/2} - \lambda^3$ As will be discussed later, the $\cos\phi$ defined in [12], we shall get $\cos\phi = -\lambda/2$ and $s_3 - s_4 = \lambda^2 A(t)$, where $A(t)$ depends on t The small expansion parameter is $\lambda \simeq 0\ 2$ and $A \simeq 0\ 9 \pm 0\ 1$ Olechowski and Poroski [8] were the first to write down the RG equations for the parameters

$$16\pi^2 \frac{d|J_{cp}|}{dt} = -3c\left(h_t^2 + h_b^2\right)|J_{cp}|,\ 16\pi^2 \frac{dA}{dt} = -\frac{3}{2}c\left(h_t^2 + h_b^2\right)A,$$

$$\frac{d\lambda}{dt} = 0,\ \frac{d\rho}{dt} = 0,\ \text{and}\ \frac{d\eta}{dt} = 0 \tag{1 17}$$

Here c = 2/3 for MSSM and J_{cp} is the irreducible phase of the CKM matrix To arrive at eq (1 17), we equate

$$A(t) = \left[\frac{M_{\text{bottom}}(t)\, M_{\text{top}}(t)}{M_{\text{top}}^2(M_X)}\right]^{-1/7} \tag{1 18}$$

Using one loop RG eqs (1 3) to (1 5), we get

$$A(M_X) = 1\ 00 \ \text{ and } A(M_Z) = 1\ 474 \tag{1 19}$$

RRR have made a complete listing and analysis They arrived at the value $\lambda = 0\ 22$ and the oft quoted results,

$$m_\tau \ m_\mu \ m_e = m_b \ m_s \ m_d = 1 \ \lambda^2 \ \lambda^4,\ m_t \ m_c \ m_u = 1 \ \lambda^4 \ \lambda^8, \tag{1 20}$$

which is very well satisfied by experimentally found masses This has been thought to be a unique and miraculeus result

We organise the paper as follows In Section 2, we shall write the RG equation with the MSSM coefficients and give the one loop exact solutions In Section 3, the unification mass for all fermions at GUT scale ranging from 113 GeV to 125 GeV as computed by Deo and Maharana [14], will be discussed as a follow up of their letters [14,19] An expression for Wolfenstein parameter in terms of RGE coefficients is given in Section 4 We propose and approximately deduce that for any fermion, other than the top $M_F = M_U \lambda^{n_F}$, where M_U = 115 MeV and n_F is an integer In Sections 5, 6 and 7, we suggest an alternative method of calculation of experimental masses of all fermions including the leptons, by a suitable self contained procedure which includes the gauge couplings as well The equation for the running of the masses of all the fermious is given in Section 8 The results are given in tables and are also shown graphically for greater clarity The concluding remarks are given in the Section 9.

2. Renormalisation group equations and solutions

As stated, the gauge sector of the standard model is characterised by three coupling constants g_3, g_2 and g_1 of $SU_C(3) \otimes SU_2(2) \otimes U_Y(1)$ respectively However these couplings are not constants, they change with energy/mass values The nature of variation is given by the solutions of the RG equations The coefficients are calculated by the specific nature of the Standard Model group For MSSM, the three couplings at mass $M_X = 2\ 2 \times 10^{16}$ GeV [15] unite to a unified coupling constant $g_U^2/4\pi \quad 1/24\ 6$ Here the supersymmetry descends from M_X down to $M_Z \doteq 91$ GeV as suggested by Witten The RG equations for the couplings in the lowest order are given by

$$16\pi^2 \frac{dg_i(t)}{dt} = c_i g_i^3(t) \qquad i = 1\ \ 2\ \ 3 \tag{2 1}$$

The coefficients are $c_1 = 6\ 6$, $c_2 = 1$, $c_3 = -3$ The first two coefficients are positive indicating that U_Y (1) and SU_L (2) are not asymptotically free, whereas the $SU_C(3)$ colour group in free and makes the entire product group asymptotically free This implies that in a perturbative formulation, the higher order contributions are small and can be neglected Therefore, we shall use eq (2 1) only in the gauge sector, with two Higgs [15]

The running parameter t is defined as $t = \log_e \mu/M_Z$ so that it varies from 0 to $\log_e(M_X/M_Z) \doteq 33$ The solution to RG equation (2 1) is

$$\frac{4\pi}{g_i^2(t)} \quad \frac{4\pi}{g_i^2} \quad \frac{}{2\pi} t \tag{2 2}$$

Here, $g_i^2 = g_i^2(0)$ are the coupling strengths in the electroweak scale M_Z Taking the value of $M_X = 2\ 2 \times 10^{16}$ GeV and $4\pi/g_U^2 = 24\ 6$, we calculate the values of $4\pi/g_1^2 = 59\ 24$, $4\pi/g_2^2 = 29\ 85$ and $4\pi/g_3^2 = 8\ 85$ These are consistent with the experimental results Thus the three coupling strengths are descendants of one coupling constant g_U

Taking the clue from eq (2 1), we rewrite the Yukawa sector SUSY RG equations given earlier in eq (1 3), which had also been written by Babu [16] following Georgi and Glashow, and Eitchen *et al* [17] in the following way with the masses in units of 175 GeV

$$16\pi^2 \frac{dM_F(t)}{dt} = A_F M_F^3(t) + \left[Y_F(t) - G_F(t)\right] M_F(t) = A_F M_F^3(t) + Z_F(t) M_F(t) \tag{2 3}$$

A_F is a theoretic factor whose value is '6' for quarks, *i e*, $F = 1, 2, \quad , 6$ and '4' for the leptons *i e*, $F = 7, 8, \quad , 12$ We have discretely taken out the main effective mass terms $M_F^3(t)$, so that we can study the equations much more carefully like the gauge theoretical calculations The way, the RGE is written above is critical and crucial The first term is like the first term of gauge sector REG (2 1) and Z_F, containing the Yukawa coupling also, has the main mass in M_F of eq (2 3) missing and this mass in M_F of eq (2 3) behaves as an external field as in some problem of quantum mechanics and is accessible for an exact integration as in eq (2 1) We repeat for ready reference that the 12 fermions are suffixed as $F = 1, 2, \quad 12$ $F = 1, 2, 3$ are the U-quarks t, c and u Similarly, $F = 4, 5, 6$ denote the D-quarks b, s and d, $F = 7, 8, 9$ are the E-leptons e μ and τ, and $F = 10, 11, 12$ are the N-neutrinos ν_e, ν_μ, ν_τ Further, $M_1 = M_{\text{top}}$, $M_2 = M_{\text{charm}}$, $\quad$, $M_{12} = M_{\text{tau}}$

A_F is a group theoretic factor whose value is '6' for quarks, *i e* $F = 1,2, \quad , 6$ and 4 for the leptons *i e* , $F = 7, 8, \quad , 12$ The positive values indicate the field theory containing Yukawa couplings only and may not be asymptotically free

Y_F is the mixing term which can be put in matrix form

$$Y_F = \sum_H A_{FH}\, M_H^\dagger(t) M_H(t), \qquad H = 1,\ 2, \quad ,\ 12 \tag{2 4}$$

In MSSM, the matrix A_{FH} is specified by the 144 elements given below,

$$A_{FH} = \begin{pmatrix} 0 & 3 & 3 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 3 & 0 & 3 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 3 & 3 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 3 & 3 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 & 0 & 3 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 3 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 3 & 3 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 3 & 3 & 3 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 & 3 & 3 & 1 & 1 & 0 & 0 & 0 & 1 \\ 3 & 3 & 3 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 3 & 3 & 3 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 3 & 3 & 3 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix} \tag{2 5}$$

The diagonal elements of A_{FH} have been taken out as the cubic term in eq (2 3), so they are zero It is illuminating to have the values in generation wise break up

Table 1. A_F for 1st generation fermions

Fermions	t	c	u	b	s	d	v	v_μ	v_e	τ	μ	e
u	3	3	6	0	0	1	1	1	1	0	0	0
d	0	0	1	3	3	6	0	0	0	1	1	1
v_e	3	3	3	0	0	0	1	1	4	0	0	1
e	0	0	0	3	3	3	0	0	1	1	1	4

Table 2. A_F for 2nd generation fermions

Fermions	t	c	u	b	s	d	v	v_μ	v_e	τ	μ	e
c	3	6	3	0	1	0	1	1	1	0	0	0
s	0	1	1	3	6	3	0	0	0	1	1	1
v_μ	3	3	3	0	0	0	1	4	1	0	1	0
μ	0	0	0	3	3	3	0	1	0	1	4	1

Table 3. A_F for 3rd generation fermions

Fermions	t	c	u	b	s	d	v	v_μ	v_e	τ	μ	e
t	6	3	3	1	0	0	1	1	1	0	0	0
b	1	0	0	6	3	3	0	0	0	1	1	1
v	3	3	3	0	0	0	4	1	1	1	0	0
τ	0	0	0	3	3	3	1	0	0	4	1	1

As the model is minimal supersymmetric, the gauge factors $G_F(t)$, which are the sum of gauge couplings, are fixed, we take the values from References [18] and [12]

$$G_U(t) = \frac{13}{15}g_1^2(t) + 3g_2^2(t) + \frac{16}{3}g_3^2(t) = \sum_{i=1}^{3} K_U^i g_i^2(t) \tag{2.6}$$

F = 1, 2, 3 stand for U and they are degenerate electromagnetic gaugewise Similarly

$$G_D(t) = \frac{7}{15}g_1^2(t) + 3g_2^2(t) + \frac{16}{3}g_3^2(t), \qquad F = 4,\ 5,\ 6 \tag{2.7}$$

$$G_E(t) = \frac{9}{5}g_1^2(t) + 3g_2^2(t), \qquad F = 7,\ 8,\ 9 \tag{2.8}$$

and

$$G_N(t) = \frac{3}{5}g_1^2(t) + 3g_2^2(t), \qquad F = 10,\ 11,\ 12 \tag{2.9}$$

Here $K_N^3 = K_E^3 = 0$, as the leptons do not have the strong colour interaction We shall need the integrals,

$$-\frac{1}{8\pi^2}\int_0^t d\tau\, G_F(\tau) = \sum_{i=1}^{3} \frac{K_i^F}{c_i} \log\left(1 - \frac{c_i g_i^2 t}{8\pi^2}\right) \tag{2.10}$$

and

$$-\frac{1}{8\pi^2}\int_0^{t_x} d\tau\, G_F(\tau) = \sum_{i=1}^{3} \frac{K_i^F}{c_i} \log\left(1 - \frac{c_i g_i^2 t_x}{8\pi^2}\right) \tag{2.11}$$

$$= \sum_{i=1}^{3} \frac{K_i^F}{c_i} \log \frac{g_i^2}{g_U^2} \tag{2 12}$$

Deo and Maharana and Deo, Maharana and Mishra [14] made a very important observation that eq (2 3) which has to be solved for a given fermion, does not contain the coefficients of the same mass in the matrix A of eq (2 5) We repeat [14] that this is the crucial and essential difference between our work and others We emphasise that $Z_F(t)$ of the RGE (2 3) does not have mixing, a term $M_F(t)$ with other fermions This is the reason that we can integrate the RGE and this is like what can be done in one loop gauge sector RGE This fact has been overlooked by all previous authors As a result, the calculational details become cumbersome and the values obtained take enormous amount of computation

Thus the present approach gives the hope of constructing a simple method of entangling the mass due to finding the solution of 12 differential equations As such, the terms $Z_F(t) = Y_F(t) - G_F(t)$ can be exponiented We introduce a subsidiary mass $m_F(t)$ through

$$M_F(t) = m_F(t) \exp\left(\frac{1}{16\pi^2} \int_0^t Z_F(\tau)\, d\tau\right), \tag{2 13}$$

such that $M_F(M_Z) = m_F(M_Z) \equiv m_F(0)$ $G_F(t)$ contains the informations about the usual gauge couplings They satisfy the equation

$$16\pi^2 \frac{dm_F(t)}{m_F^3} = A_F \exp\left(\frac{1}{8\pi^2} \int_0^t Z_F(\tau)\, d\tau\right) dt \tag{2 14}$$

They look, astonishingly, similar to the gauge sector one loop RG eq (2 1) and can be solved exactly Integrating eq (2 14) from M_Z to M_X *i e* , from t = 0 to t_X, we get

$$\frac{8\pi^2}{m_F^2(M_Z)} = \frac{8\pi^2}{m_F^2(M_X)} + A_F \int_0^{t_X} dt \exp\left(\frac{1}{8\pi^2} \int_0^t Z_F(\tau)\, d\tau\right) \tag{2 15}$$

Putting back the exponential,

$$\frac{M_{\rm top}^2(M_Z)}{M_F^2(M_Z)} = \frac{M_{\rm top}^2(M_Z)}{M_F^2(M_Z)} \exp\left(\frac{1}{8\pi^2} \int_0^{t_X} Z_F(\tau)\, d\tau\right) + \frac{A_F}{8\pi^2} \int_0^{t_X} dt \exp\left(\frac{1}{8\pi^2} \int_0^t Z_F(\tau)\, d\tau\right) \tag{2 16}$$

This is the exact one loop solution $M_F^2(M_Z)$ is the mass of the fermions at M_Z

The descent or ascent running of the masses from GUT M_X to electroweak M_Z, can be obtained by integrating eq (2 14) from $t = t_X$ to t The result which is not given in Reference [14] is

$$\frac{8\pi^2 M_{top}^2}{M_F^2(t)} = 8\pi^2 \frac{M_{top}^2}{M_F^2(M_X)} \exp\left(\frac{1}{8\pi^2} \int_t^{t_x} Z_F(\tau)\, d\tau\right) + A_F \int_t^{t_x} dt_1 \exp\left(\frac{1}{8\pi^2} \int_t^{t_1} Z_F(\tau)\, d\tau\right) \quad (2\ 17)$$

This is also one loop exact By solving eqs (2 16) and (2 17), we can find M_F (M_X) and $M_F(t)$, respectively

3. Original mass of all fermions at M_X

The gauge integrals over $G_F(t)$ are easily and acurately calculable The most difficult task is to evaluate $Y_F(t)$ Even though, it does not contain $M_F(t)$, it is a sum of squares of moduli of the masses of all fermions For example, from the matrix as given by eq (2 5),

$$Y_{top}(t) = 3M_c^2(t) + 3M_u^2(t) + M_b^2(t) + M_{\nu_\mu}^2(t) + M_{\nu}^2(t) + M_{\nu}^2(t) \quad (3\ 1)$$

There is mixing of six other fermions for the top Since the top is heaviest of all fermions we take the mass of top as input in our calculation

In the first approximation, one retains only those terms containing $M_{top}^2(t)$ occurring in any integral with $Y_F(t)$ Consider the top case $Y_{top}(t)$ can be set equal to zero in the RG equation for the top Then $G_F(t)$ is the whole plain Yukawa contribution The top mass is then given by a simple expression

$$1 = \frac{M_{top}^2(M_Z)}{M_{top}^2(M_X)} C_{top} + D_{top}, \quad (3\ 2)$$

where, using integrals (2 10) to (2 12),

$$C_{top} = \prod_{i=1}^{3} \left(\frac{g_i^2}{g_U^2}\right)^{K_c^F} = 0\ 086 \text{ and}$$

$$D_{top} = \frac{6}{8\pi^2} \int_0^{t_X} {}^{33} dt \prod_{i=1}^{3} \left(1 - \frac{g_i^2 c\, t}{8\pi^2}\right)^{K_c^F} = 0\ 802 \quad (3\ 3)$$

Putting these values, the original mass of the top, at an energy of $2\ 2 \times 10^{16}$ GeV was, approximately,

$$M_{top}(M_X) = M_{top}(M_Z)\left(1 - D_{top}\right)^{1/2} C_{top}^{1/2} = 114 \text{ GeV} \quad (3\ 4)$$

This is Deo-Maharana's [14] result We shall take the value of the mass of top quark to be 115 GeV which is at least a best approximation at the scale M_X

We can write the general formula for all the fermions, using the top mass as

$$\frac{M_{\text{top}}^2(M_Z)}{M_F^2(M_Z)} = \frac{M_{\text{top}}^2(M_Z)}{M_F^2(M_X)} C_F + D_F, \tag{3 5}$$

where

$$C_F = \prod_{i=1}^{3} \left(\frac{g_i^2}{g_U^2} \right)^{K_c^F} \exp\left(\frac{1}{8\pi^2} \int_0^{t_X} Y_F(\tau)\, d\tau \right) \tag{3 6}$$

and
$$D_F = \frac{A_F}{8\pi^2} \int_0^{t_X} dt \prod_{i=1}^{3} \left(1 - \frac{c_i g_i^2 t}{8\pi^2} \right)^{K_c^F} \exp\left(\frac{1}{8\pi^2} \int_0^t Y_F(\tau)\, d\tau \right) \tag{3 7}$$

It is not so easy to calculate Y_F for other fermions even with the approximation given by eq (1 6) So we consider the Yukawa-like coupling $h_F(t)$ for $F = 2, 3, \ldots, 12$, which is related to $M_F(t)$ as

$$M_F(t) = h_F(t) \exp\left(-\frac{1}{16\pi^2} \int_0^t G_F(\tau)\, d\tau \right) \tag{3 8}$$

Here we have not taken the top The RG equation for $h_F(t)$ is,

$$8\pi^2 d\left(\log h_F^2(t)\right) = A_F h_F^2(t) \exp\left(-\frac{1}{8\pi^2} \int_0^t G_F(\tau)\, d\tau \right) dt + Y_F(t)\, dt \tag{3 9}$$

Here $h_F(t)$ contains the usual β-angle factors of the Higgs system As has been discussed in Section 1, we may not need this angle in our approach to the problem at hand If $F > 1$, the heaviest fermion next to the top is the bottom The first term of eq (3 9), at the M_Z scale, is 1/1235 times smaller, whereas the last term contains one h_{top} in h_{bottom} To a good approximation, and to begin with, we shall neglect this term and express $h_F(t)$ in terms of $Y_F(t)$'s Then

$$\int_0^{t_X} Y_F(\tau) d\tau = 8\pi^2 \log \frac{h_F^2(M_X)}{h_F^2(M_Z)} \tag{3 10}$$

Here, $h_F^2(M_Z) = M_F^2(M_Z)$ by definition In the units of top mass 175 GeV,

$$h_F^2(M_X) = M_F^2(M_X) \exp\left(-\frac{1}{8\pi^2} \int_0^{t_X} G_F(\tau)\, d\tau \right) = M_{\text{top}}^2(M_X) \exp\left(\frac{1}{8\pi^2} \int_0^{t_X} G_F(\tau)\, d\tau \right) \tag{3 11}$$

$$= M_{\text{top}}^2(M_Z) = 1. \tag{3 12}$$

So,

$$\int_0^{t_x} Y_F(\tau)\,d\tau = -8\pi^2 \log M_F^2(M_Z) \text{ or } \exp\left(\frac{1}{8\pi^2}\int_0^{t_x} Y_F(\tau)\,d\tau\right) = \frac{M_{top}^2(M_Z)}{M_F^2(M_Z)} \quad (3\ 13)$$

This is true as long as the first term of eq (3 9) is negligible Since there is M_F^3 in this term, it is much more justifiable to have $D_{lepton} = 0$ The general eq (3 5) for the leptons gives

$$\frac{M_{top}^2(M_Z)}{M_{lepton}^2(M_Z)} = \frac{M_{top}^2(M_Z)}{M_{lepton}^2(M_Z)} C_{lepton} \quad (3\ 14)$$

where

$$C_{lepton} = \prod_{i=1}^{3}\left(\frac{g_i^2}{g_U^2}\right)^{\frac{K^{lepton}}{c_i}}, \text{ giving } M_{lepton}^2(M_X) = M_{lepton}^2 C_{lepton}$$

Putting the gauge constants and masses in the above, as experimentally reported, we get the masses at GUT scale for different leptons as given in Table 4

Table 4. Unification scale mass for leptons

Lepton	Unification scale mass (GeV)
e	116
μ	116
τ	116
ν_e	126
ν_μ	126
ν	126

At the grand unification mass, the electron, the muon and the tau lepton climb to 116 GeV in this approximation The calculation for the neutrinos are not reliable as the isospin factors are uncertain and may be inaccurate

We are now left with the remaining five quarks For them, we attempt to find the next leading order approximation, *i e*, we first set the first term equal to zero and obtain

$$\exp\left(\frac{1}{8\pi^2}\int_0^{t_X} Y_Q(\tau)d\tau\right) = \frac{1}{h_Q^2(M_Z)} = \frac{M_{top}^2(M_Z)}{M_Q^2(M_Z)} \quad (3\ 15)$$

This is used in the calculation for $C_F = C_Q$ of eq (3 6), which gives

$$C_Q = \prod_{i=1}^{2}\left(\frac{g_i^2}{g_U^2}\right)^{\frac{K_i^Q}{C_i}} \frac{1}{M_Q^2(M_Z)}, \exp\left(\frac{1}{8\pi^2}\int_0^t Y_F(\tau)d\tau\right) = \frac{h_Q^2(t)}{M_Q^2(M_Z)}, \tag{3 16}$$

and

$$D_Q = \frac{6}{8\pi^2}\int_0^{t_X} \frac{h_Q^2(t)}{M_Q^2(M_Z)}\prod_{i=1}^{2}\left(1-\frac{g_i^2 c_i t}{8\pi^2}\right)^{\frac{K_i^Q}{C_i}} dt \tag{3 17}$$

We look for ways to calculate $h_Q(t)$ We can retain only the top quark in the RG equation in a slightly different way than what has been taken by RRR [6] and get,

$$8\pi^2 d\log h_Q^2(t) \simeq N_Q dt \tag{3 18}$$

We have, from this result, read out as

$$N_{\text{charm}} = 3,\ N_{\text{up}} = 3,\ N_{\text{bottom}} = 1,\ N_{\text{strange}} = N_{\text{down}} = 0 \tag{3 19}$$

To use this value in Y_F, it is necessary to take an average as the couplings change rapidly,

$$\overline{\log h_Q^2(t)} = \frac{1}{t_X}\int_0^t \frac{d}{d\tau}\log h_Q(\tau)d\tau = \frac{1}{t_X}\int_0^t \frac{N_Q}{8\pi^2}d\tau = \frac{t}{t_X}\frac{N_Q}{8\pi^2},$$

$$\text{or}\quad \overline{h_Q^2(t)} = \exp\left(\frac{t}{t_X}\frac{N_Q}{8\pi^2}\right) \tag{3 20}$$

This expression for Yukawa coupling, is not much different from unity for all allowed t's as has been stated before We arrive at the following result,

$$D_Q = \frac{6}{8\pi^2}\int_0^{t_X} dt \exp\left(\frac{t}{t_X}\frac{N_Q}{8\pi^2}\right)\prod_{i=1}^{2}\left(1-\frac{g_i^2 c_i t}{8\pi^2}\right)^{\frac{K_i^Q}{C_i}} \tag{3 21}$$

The unification mass is calculated numerically from

$$M_Q(M_X) = M_{\text{top}}\left(\frac{C_Q}{1-D_Q}\right)^{1/2} \tag{3 22}$$

The results, for the quarks, are given in Table 5 We note that $M_{\text{charm}}(M_X) = M_{\text{up}}(M_X) \neq M_{\text{top}}(M_X)$ and $M_{\text{strange}}(M_X) = M_{\text{down}}(M_X) \neq M_{\text{bottom}}(M_X)$

Table 5. Unification scale mass for quarks

Quark	Unification scale mass (GeV)
Top	114
Charm	115
Up	115
Bottom	119
Strange	118
Down	118

Thus, we have shown that all fermions seem to originate at an energy 2 2 × 10^{16} GeV with approximate mass of about 115 GeV Perhaps, this is due to the equality of $A \simeq 1$ in Wolfenstein's parametrisation of CKM matrix In this perturbative method of solution for finding the unification mass, information about the masses of the 11 fermions has been lost due to cancellation of $M^2(M_Z)$ in both r h s and *l* h s of the eq (2 16) due to use of eqs (3 13) and (3 17) The result of a common mass at the origin which we have shown to exist is, at least a hypothesis to pursue further

4. Deduction of Wolfenstein parameter and the rotational integers

As the original equation shown explicitly the top mass decreases with energy but all the other quarks and leptons, starting from the value at M_X acquire smaller and smaller values and become quite light in the electroweak scale The descent or ascent eq (2 17) describing the 'run' can be put in a form like eq (3 5)

$$\frac{M_{\text{top}}^2(M_Z)}{M_F^2(t)} = \frac{M_{\text{top}}^2(M_Z)}{M_F^2(M_Z)} C_F(t) + D_F(t), \tag{4 1}$$

where
$$C_F(t) = a_F(t) \exp\left(\frac{1}{8\pi^2} \int_0^{t_X} Y_F(\tau) d\tau \right), \tag{4 2}$$

$$D_F(t) = \frac{A_F}{8\pi^2} \int_t^{t_X} dt_1 b_F(t_1) \exp\left(\frac{1}{8\pi^2} \int_t^{t_1} Y_F(\tau) d\tau \right), \tag{4 3}$$

$$a_F(t) = \prod_{i=1}^{3} \left[\frac{\left(1 - \frac{g_i^2 c_i t_X}{8\pi^2} \right)}{1 - \frac{g_i^2 c_i t}{8\pi^2}} \right]^{\frac{K_i^F}{c_i}}, \tag{4 4}$$

and
$$b_F(t)=\prod_{i=1}^{3}\left[\frac{\left(1-\dfrac{g_i^2c_it_1}{8\pi^2}\right)}{1-\dfrac{g_i^2c_it}{8\pi^2}}\right]^{\frac{K_i^F}{c_i}} \tag{4 5}$$

As an example, let us take the case of top For the top, we can take $Y_F \to 0$ and calculate variation of its mass from $M_U \simeq 115$ GeV to the top mass 175 GeV

$$M_{\text{top}}(t)=\frac{M_{\text{top}}(M_X)}{\left(C_{\text{top}}(t)+\dfrac{M_U^2}{M_{\text{top}}^2}D_{\text{top}}(t)\right)^{1/2}} \tag{4 6}$$

The values are given in Table 8 of Section 8

It has not been possible to get any reliable results for other fermions Let us try another method For other cases than the top, we shall try to fit them into a scheme which is not only much simpler, but has better physical content In the following we discuss a different route for solutions without specifying gauge factors completely These give much better information about the mass value for all the fermions For other cases than top, let us construct a function $B_F(t)$ such that

$$8\pi^2\frac{d}{dt}\log B_F^2(t)=Z_F(t) \tag{4 7}$$

where $Z_F(t)=Y_F(t)-G_F(t)$ contains the gauge factors in $G_F(t)$ also Then

$$\exp\left(\frac{1}{8\pi^2}\int_0^t Z_F(\tau)d\tau\right)=\frac{B_F^2(t)}{B_F^2(M_Z)}, \tag{4 8}$$

and
$$\exp\left(\frac{1}{8\pi^2}\int_{t_X}^t Z_F(\tau)d\tau\right)=\frac{B_F^2(t)}{B_F^2(M_X)} \tag{4 9}$$

Eq (2 16) reduces to

$$M_F(M_Z)=\frac{\dfrac{B_F(M_Z)}{B_F(M_X)}M_F(M_X)}{\left(1+\dfrac{M_F^2(M_X)}{M_{\text{top}}^2}\dfrac{A_F}{8\pi^2}\displaystyle\int_0^{t_X}dt\,\frac{B_F^2(t)}{B_F^2(M_z)}\right)^{1/2}} \tag{4 10}$$

As indicated in the preceeding section, we neglect the terms with A_F for all quarks except the top Then

$$M_F(M_Z) \simeq M_F(M_X)\frac{B_F(M_Z)}{B_F(M_X)} - M_F(M_X)\exp\left(-\frac{I_F}{16\pi^2}\right), \tag{4 11}$$

where the integral I_F is

$$I_F = \int_0^{t_X} Z_F(t)dt = \int_0^{t_X}\left(\sum_G A_{FG} M_G^\dagger(t) M_G(t) - G_F(t)\right)dt, \tag{4 12}$$

with A_{tG} = 0 to indicate that this is not for the top

The purpose of introducing the logarithms in eq (4 7) is to incorporate the possible multivaluedness of I_F From eq (4 12),

$$I_F = \int_0^{t_X} Z_F(t)dt = \int_0^{t_X}\left(\sum_G A_{FG} M_G^\dagger(t) M_G(t) - G_F(t)\right)dt \tag{4 13}$$

$$= \frac{1}{2}\int_0^{t_X}\left(\sum_G A_{FG} M_G^\dagger(t) M_G(t) - G_F(t) + \sum_G A_{FG} M_G^\dagger(-t) M_G(-t) - G_F(-t)\right)dt \tag{4 14}$$

$$= \frac{1}{4}\int_{-t_X}^{t_X} \frac{dt}{dM_F} dM_F(t)\left(\sum_G A_{FG} M_G^\dagger(t) M_G(t) - G_F(t) + \sum_G A_{FG} M_G^\dagger(-t) M_G(-t) - G_F(-t)\right) \tag{4 15}$$

$$= \frac{16\pi^2}{4}\int_{-M_U}^{M_U} \frac{dM_F(t)}{M_F(t)} \frac{\left(\sum_G A_{FG} M_G^\dagger(t) M_G(t) - \frac{1}{2}(G_F(t)+G_F(-t))\right)}{\left(A_{FF} M_F^\dagger(t) M_F(t) - G_F(t)\right)}, \tag{4 16}$$

where we have used eq (2 3) There are poles in the integrand as can be noticed Let us set

$$M_F(t) = M_U e^{i\theta_F(M_F(t))n_F} = M_U e^{i\theta_F(t)n_F},$$

where n_F is the integer and specifying a particular Riemann sheet M_U is the GUT scale mass relation We will call it the rotational integer since $t = M_U e^{i\theta_F(t)n_F}$ maps the complex t-plain to inside of a circle of radius M_U Using the above in eq (4 16), we get

$$I_F = in_F \frac{16\pi^2}{2} \oint d\theta_F(t) \frac{\Sigma_G A_{FG} - \frac{G_F(t)}{M_U^2}}{A_F + \Sigma_G A_{FG} - \frac{G_F(t)}{M_U^2}} \tag{4 17}$$

There are poles in the integrand depending on M_U and G_F We shall omit the inconsequential factor G_F/M_U^2 in the numerator We note that for quarks $A_F + \Sigma_{G\ 1}^6 = 6 + 7 = 13$ and for leptons 4 + 9 = 13 The integral is almost a constant We isolate the integer n_F, characterising F from the integral, using $1 = (1/12)\Sigma_H$ in eq (4 17) then find

$$I_F = in_F \frac{16\pi^2}{2} \frac{1}{12} \sum_H \sum_G A_{HG} \int_{-t_X}^{t_X} d\theta_H(t) \frac{1}{A_H + \Sigma_H A_{HG} - \frac{G_H}{M_U^2}} \tag{4 18}$$

In the above we have averaged over the twelve fermions Retracing the steps and using $M_H(t) = M_U e^{i\theta_H(t) n_F}$ in eq (4 18), we finally get

$$I_F \simeq n_F \frac{1}{2} \sum_H \sum_G A_{HG} \int_0^{t_X} dt = \frac{1}{12} n_F t_X \sum_H \sum_G A_{HG} \tag{4 19}$$

First, we shall be interested in the CKM matrix for quarks with lepton masses taken as zero Then let G vary from 1 to 6, whereas F will be taking values from 2 to 12, since all of them contains 6 quarks Only the coefficients A_{HG} are needed to calculate I_F of eq (4 19) The coefficients A_{FG} have non-vanishing values for F = 2, 3, , 12 and G = 1 2 , 6

$$\text{For } F = 2, \ldots, 6, \qquad A_{F1} + A_{F2} + \cdots + A_{F6} = 7, \tag{4 20}$$

and

$$\text{For } F = 7, \ldots, 12, \qquad A_{F1} + A_{F2} + \cdots + A_{F6} = 9 \tag{4 21}$$

These are obtained from all the numbers given in eq (2 5) and in Tables 1, 2 and 3 Care should be taken to calculate these values from the corresponding Tables For the twelve fermions, the sum of the values of the coefficients A_{FG} is (7 × 5) + (9 × 6) = 89 The average of Z, *i e*, $\tilde{Z}$ is given

$$\tilde{Z} = 89/12, \qquad \text{and} \qquad I_F = n_F\, t_X \frac{89}{12} \tag{4 22}$$

The masses of all the fermions due to quark-lepton equivalence other than the top is

$$M_F(M_Z) \simeq M_F(M_Z)\, e^{-n_F \frac{t_X}{16\pi^2} \frac{89}{12}} \simeq \lambda^{n_F} M_U \tag{4 23}$$

where λ_{n_F} is λ raised to the power of $\left(t_X/16\pi^2\right)(89/12)$ The Wolfenstein parameter λ is readily obtained by letting n_F = 1 in the exponential for the ratio of the masses,

$$\lambda = \exp\left(-\frac{t_X}{16\pi^2}\frac{89}{12}\right) = 0\ 219 \tag{4 24}$$

This is an excellent result in spite of the approximate estimates

The Table 6 identifies the particles We have increased n_F by neighbourhood integers which we have called the rotational integers

Table 6 Identification of fermions

n_F	Mass in GeV	Fermion	Expt Result (GeV)
2	5 5	bottom (b)	5
3	1 2	charm (c), tau lepton (τ)	1 4 1 7
4	0 264	strange (s) muon (μ)	0 1–0 23 0 01
6	0 012	down (d) tau neutrino (ν)	0 055–0 115
7	0 0028	up (u)	0 003
8	6×10^{-4}	electron (e)	$\times 10^{-4}$
9	$1\ 33 \times 10^{-4}$	muon neutrino (ν_μ)	$1\ 9 \times 10^{-4}$
16	3×10^{-9}	electron neutrino (ν_e)	3×10^{-9}

This, by itself, is a very notable equation for phenomenology This is deduced in a different context and arguments than those given in normal circumstances The introduction of λ^{n_F} is one such device to put the various fermions in the top integral mass source, from which they are deduced The method of approach is different from the usual ones But the results should yield the values observed experimentally The following analysis is entirely new and gives the Table 6 The values are still approximate and we shall try to improve them as we proceed We also remark, it may not be necessary for the calculations we shall perform below but provides a good guideline to go further into the details of all the calculations

5. Gauge contributions

5 1 The masses of top, charm and up quarks

It is easy to write down the equation for the top mass in terms of the mass $M_F(M_X)$ and gauge couplings

$$M_{\text{top}} = M_{\text{top}}(M_X)\left(\frac{1-d_{\text{top}}}{a_{\text{top}}}\right)^{1/2} \tag{5 1}$$

We shall also need the equation for the descent of top mass

$$\frac{M_{\text{top}}^2}{M_{\text{top}}^2(t)} = \frac{M_{\text{top}}^2}{M_{\text{top}}^2(M_X)} a_{\text{top}}(t) + D_{\text{top}}(t) \tag{5 2}$$

The equation contains only the gauge factors This agrees with our previous equation for the top and the nature of variation with '*t*' should be more carefully be noted

5 2 The masses of charm and up quarks

The RG equation for the top is non-linear and intermixed Specifically it is

$$8\pi^2 \frac{d}{dt} \log M_{\text{top}}^2(t) = -G_U + 6M_{\text{top}}^2(t) + Y_{\text{top}}(t) \tag{5 3}$$

Following a texture analysis of the RG solutions, we introduce another function $B(t)$, (not to be confused with $B_F(t)$), which satisfies

$$8\pi^2 \frac{d}{dt} \log B^2(t) = -G_U + 6M_{\text{top}}^2(t) \tag{5 4}$$

Eq (5 4) does not specify the function $B(t)$ completely except that $d \log B(t) = d \log (M_{\text{top}}(t))$ This lone restriction gives us an infinite number of free choices Because letting $B \to \xi B$, where ξ is an arbitrary constant, does not change the top mass function $M_{\text{top}}(t)$ For simplicity, we take the second derivative of this function $B(t)$ to be zero so that

$$d \log B(t) = C_B dt \tag{5 5}$$

C_B is essentially $d \log M_{\text{top}}(t), M_{\text{top}}(t)$ changes by only 30 to 50 GeV as the mass μ goes from 91 GeV to 2 2 × 10^{16} GeV $C_B \simeq (175-(123 \text{ to } 115))/175 \simeq 0\,297$ to 0 342 We take C_B = 0 3 The nonequiness due to nonlinearity can also be seen as follows

$$d \log B(t) = d \log M(t) = d \log \frac{M_{\text{top}}}{M_o} = \frac{dt}{16\pi^2}\left[-G_U + 6\left(\frac{M_{\text{top}}}{M_o}\right)^2\right]$$

M_o is an arbitrary constant We can choose M_o suitably so that

$$\left[-G_U + 6\left(\frac{M_{\text{top}}}{M_o}\right)^2\right] \Big/ 16\pi^2 \simeq 0\,3 = C_B$$

Integrating from $t = t_X(M_X)$ to $t = t_X(M_Z)$, we have

$$\log \frac{B_Z}{B_U} = -C_B t_X, \quad \text{and} \quad \frac{B_Z}{B_U} = e^{C_B t_U} = (0.192)^6. \tag{5.6}$$

Furthermore, integrating from $t = t_X$ to arbitrary t,

$$\frac{B(t)}{B_U} = \exp\left(C_B(t - t_X)\right) = \left[\left(\frac{B_Z}{B_U}\right)^{1/6}\right]^{(1-t/t_X)} \tag{5.7}$$

Let us examine the case for the charm. We have

$$8\pi^2 \frac{d}{dt} \log M_c^2(t) = -G_U + 3M_t^2 = -G_U + \frac{1}{2}\left[6M_t^2 - G_U\right] + \frac{1}{2} G_U$$

$$= -\frac{1}{2} G_U + 4\pi^2 \frac{d}{dt} \log B^2(t). \tag{5.8}$$

Integrating from t_X to t,

$$\log \frac{M^2_{\text{charm}}(t)}{M^2_{\text{charm}}(M_X)} = \frac{1}{2} \log \frac{B^2(t)}{B_U^2} - \frac{1}{16\pi^2} \int_{t_X}^{t} G_U(t) dt, \tag{5.9}$$

$$M_{\text{charm}}(t) = M_U \left(\frac{B(t)}{B_U}\right)^{1/2} \exp\left(-\frac{1}{32\pi^2} \int_{t_X}^{t} G_U(\tau) d\tau\right), \tag{5.10}$$

$$M_{\text{charm}}(M_Z) = M_U \left(\frac{B_Z}{B_U}\right)^{1/2} C_{\text{top}}^{-1/4}, \quad C_{\text{top}}^{-1/4} = 1 \cdot 8466,$$

and

$$M_c = \left[\left(\frac{B_Z}{B_U}\right)^{1/6} C_{top}^{-1/12}\right]^3 M_{\text{charm}}(M_X). \tag{5.11}$$

From Section 4, we now calculate

$$\lambda_{\text{charm}} = e^{-C_B t_X/6} C_{\text{top}}^{-1/12} = 0.221 \tag{5.12}$$

Again this is a good result. The mass of the charm is 1.24 GeV from the eq. (5.11) in very good agreement with the experimental value. The rotational integer n_F is three as in Table 6.

We continue further and consider the up quark The 'heavy top integral' approximation of the RGE for the up quark is

$$8\pi^2 \frac{d}{dt} \log M_{\text{up}}^2 = -G_U + 3M_{\text{top}}^2(t), \tag{5 13}$$

Guessing from the general rotational integral parametrization, as shown in Table 6 as λ^7 we write this equation as

$$8\pi^2 \frac{d}{dt} \log M_{\text{up}}^2 = -G_U + \frac{1}{2}\left(6M_{\text{top}}^2(t)\right),$$

$$= -\frac{G_U}{2} + 8\pi^2 \frac{d}{dt} \log B^2(t) - \frac{1}{2} 8\pi^2 \frac{d}{dt} \log M_{\text{top}}^2(t) \tag{5 14}$$

Integrating, we get

$$\frac{M_{\text{up}}}{M_U} = \left(\frac{B_Z}{B_u}\right)\left(\frac{M_U}{M_{\text{top}}}\right)^{1/2} a_U^{1/4}, \tag{5 15}$$

$$\lambda_{\text{up}} = \left[\left(\frac{B_Z}{B_u}\right)\left(\frac{M_U}{M_{\text{top}}}\right)^{1/2} a_U^{-1/4}\right]^{1/7} = 0\,220 \tag{5 16}$$

This gives $M_{up} \simeq 0\,0029$ GeV, with $M_U = M_{\text{top}}(M_X)$, a little lower value but quite close to the experimental result The values depend crucially on C_B, which is itself not exact It is also true that M_U may be a little different from 115 GeV

6. Masses of the down quarks

The calculation of the mass of up-quark has set the method of finding solutions close to the experimental values by using the texture analysis function $B(t)$ ignoring the terms with coefficients A_F We present a general recipe from the 'heavy top integral' approximation Let the RG equation for any $F \neq 1$ be

$$8\pi^2 \frac{d}{dt} \log M_F^2 = -G_F + N_F M_{\text{top}}^2 \tag{6 1}$$

Here N_F can be zero as well

$$N_F M_{\text{top}}^2(t) = \frac{N_F}{6}\left(6M_{\text{top}}^2(t) - G_U\right) + \frac{N_F}{6} G_U,$$

$$= \frac{N_F + \eta_F}{6}\left(8\pi^2 \frac{d}{dt}\log B^2(t)\right) - \frac{\eta_F}{6}\left(8\pi^2 \frac{d}{dt}\log M_{\rm top}^2(t)\right) + \frac{N_F}{6} G_U \tag{6 2}$$

η_F can be determinable coefficient All of them will satisfy the RG equation because of eq (5 4) One may wonder how the top can enters into the picture everywhere The reason is that the average integrals for all the quarks as noted in eq (3 20) is nearly equal to one B works like a spectrator But they should let n_F and λ be such that they are within rotational integers and gauge interaction contributions

Integrating from the known values, t_X to 0, we get

$$M_F(M_Z) = M_U \exp\left(-\frac{1}{16\pi^2}\int_0^{t_X} G_F(t)dt - \frac{N_F}{96\pi^2}\int_0^{t_X} G_U(t)dt\right)\left(\frac{B_Z}{B_U}\right)^{\frac{N_F+\eta_F}{6}}\left(\frac{M_U}{M_{\rm top}}\right)^{\frac{\eta_F}{6}} \tag{6 3}$$

Using values of η_F, which are easily determined and fairly approximately calculated For the down quarks,

$$8\pi^2 \frac{d}{dt}\log M_b^2 = -G_D + M_{\rm top}^2 \tag{6 4}$$

$$8\pi^2 \frac{d}{dt}\log M_{s\,d}^2 = -G_D \tag{6 5}$$

So $$8\pi^2 \frac{d}{dt}\log M_b^2 = -G_D + 2M_{\rm top}^2 - M_{\rm top}^2, \tag{6 6}$$

$$= -G_D + \frac{25}{6}\left(6M_{\rm top}^2 \quad G_U\right) - \frac{15}{6}\left(6M_{\rm top}^2 - G_U\right) + \frac{G_U}{6}, \tag{6 7}$$

$$= -G_D + \frac{25}{6}\left(8\pi^2 \frac{d}{dt}\log B^2\right) - \frac{15}{6}\left(8\pi^2 \frac{d}{dt}\log M_{\rm top}^2\right) + \frac{G_U}{6} \tag{6 8}$$

Integrating from t_X to 0, we get

$$\frac{M_b}{M_U} = \left(\frac{B_Z}{B_U}\right)^{25/6}\left(\frac{M_U}{M_{\rm top}}\right)^{15/6} a_D^{-1/2} a_U^{1/12}, \tag{6 9}$$

$$= \left[\left(\frac{B_Z}{B_U}\right)^{25/12}\left(\frac{M_U}{M_{\rm top}}\right)^{15/12} a_D^{-1/4} a_U^{1/24}\right]^2 \tag{6 10}$$

The quantity in the square bracket is $\lambda \simeq 0.203$. This gives the value of the bottom mass as to be 4.7 GeV.

For the strange we have,

$$8\pi^2 \frac{d}{dt}\log M_s^2 = -G_D + \frac{4.5}{6}\left(6\pi^2 \frac{d}{dt}\log B^2 - G_U\right) - \frac{4.5}{6}\left(6\pi^2 \frac{d}{dt}\log M_{\text{top}}^2 - G_U\right), \quad (6\ 11)$$

This leads to

$$\frac{M_s}{M_U} = \left(\frac{B_Z}{B_U}\right)^{4\ 5/6}\left(\frac{M_U}{M_{\text{top}}}\right)^{4\ 5/6} a_D^{-1/2} = \left[\left(\frac{B_Z}{B_U}\right)^{4.5/24}\left(\frac{M_U}{M_{\text{top}}}\right)^{4\ 5/24} a_D^{-1/8}\right]^4 = \lambda^4. \quad (6\ 12)$$

Here λ comes out to be near 0.196 and the strange mass is found to be 0.168 GeV

Proceeding further to the down quark, we get

$$8\pi^2 \frac{d}{dt}\log M_d^2 = -G_D + \frac{6.5}{6}\left(6\pi^2 \frac{d}{dt}\log B^2 - G_U\right) - \frac{6.5}{6}\left(6\pi^2 \frac{d}{dt}\log M_{\text{top}}^2 - G_U\right), \quad (6\ 13)$$

which gives

$$\frac{M_d}{M_U} = \left(\frac{B_Z}{B_U}\right)^{6\ 5/6}\left(\frac{M_U}{M_{\text{top}}}\right)^{6\ 5/6} a_D^{-1/2} = \left[\left(\frac{B_Z}{B_U}\right)^{6\ 5/36}\left(\frac{M_U}{M_{\text{top}}}\right)^{6\ 5/36} a_D^{-1/12}\right]^6 \simeq \lambda^6. \quad (6\ 14)$$

This gives $\lambda \simeq 0.19$ and M_s = 0.0053 GeV. The ratio $M_s/M_d \simeq 31$. However, from eq (1.20), this ratio is

$$\frac{M_s}{M_d} = \lambda^{-2} = (0.2)^{-2} = 25. \quad (6\ 15)$$

The experimental values are close to 25.

7. The masses of the leptons

Ignoring the non-linear terms proportional to A_F = 4, the RG equation, with the mass of the top quark only, for the six leptons are

$$8\pi^2 \frac{d}{dt}\log M_{e,\mu,\tau}^2 = -G_E, \quad (7.1)$$

$$8\pi^2 \frac{d}{dt}\log M^2_{v_e\, v_\mu\, v_\tau} = -G_N + 3M^2_{\rm top} \tag{7 2}$$

For the first three electron-leptons, we write, with choices based on $\lambda \simeq 0\ 22$ and rotational integer n_F,

$$8\pi^2 \frac{d}{dt}\log M^2_{e\,\mu\,\tau} = -G_E + \left(\frac{30}{24},\frac{17}{24},\frac{11}{24}\right)\left(8\pi^2 \frac{d}{dt}\log B^2(t)\quad G_U\right)$$

$$-\left(\frac{30}{24},\frac{17}{24},\frac{11}{24}\right)\left(8\pi^2 \frac{d}{dt}\log M^2_{\rm top}(t)\quad G_U\right), \tag{7 3}$$

and obtain

$$\frac{M_e}{M_U} = \left(\frac{B_Z}{B_U}\right)^{5/4}\left(\frac{M_U}{M_{\rm top}}\right)^{5/4} a_E^{-1/2} = \left[\left(\frac{B_Z}{B_U}\right)^{5\ 32}\left(\frac{M_U}{M_{\rm top}}\right)^{5\ 32} a_E^{1/16}\right]^8 = \lambda_e^8, \tag{7 4}$$

which gives $\lambda_e = 0\,209$, $M_e = 4 \times 10^{-4}$ GeV Similarly

$$\frac{M_\mu}{M_U} = \left(\frac{B_Z}{B_U}\right)^{17\ 24}\left(\frac{M_U}{M_{\rm top}}\right)^{17\ 24} a_E^{1/2} = \left[\left(\frac{B_Z}{B_U}\right)^{17\ 120}\left(\frac{M_U}{M_{\rm top}}\right)^{17\ 120} a_E^{1/10}\right]^5 = \lambda_\mu^5, \tag{7 5}$$

which gives $\lambda_\mu = 0.238$, $M_\mu = 0.09$ GeV,

and

$$\frac{M_\tau}{M_U} = \left(\frac{B_Z}{B_U}\right)^{11/24}\left(\frac{M_U}{M_{\rm top}}\right)^{11,24} a_E^{-1/2} = \left[\left(\frac{B_Z}{B_U}\right)^{11/72}\left(\frac{M_U}{M_{\rm top}}\right)^{11/72} a_E^{1/6}\right]^3 = \lambda_\tau^3, \tag{7 6}$$

which gives $\lambda_\tau = 0\,23$, $M_\tau = 136$ GeV

The masses of the neutrinos have not yet been measured, only limits have been set The reported values, as shown in Table 6, fall into a good pattern, namely, $M_{v_e} = M_U\lambda^{16}$, $M_{v_\mu} = M_U\lambda^9$ and $M_{v_\tau} = M_U\lambda^6$, $\lambda = 0.22$ So the equivalent, degeneracy lifting equations, showing the choices of n_F are

$$8\pi^2 \frac{d}{dt}\log M^2_{v_e} = -G_N + \frac{5}{2}\left(6M^2_{\rm top} - G_U\right) - 2\left(6M^2_{\rm top} - G_U\right) + \frac{1}{2}G_U,$$

$$= -G_N + \frac{5}{2}\left(8\pi^2 \frac{d}{dt}\log B^2\right) - 2\left(8\pi^2 \frac{d}{dt}\log M_{\text{top}}^2\right) + \frac{1}{2}G_U, \tag{7.7}$$

$$8\pi^2 \frac{d}{dt}\log M^2_{\nu_\mu \nu_\tau} = -G_N + \left(\frac{3}{2}, 1\right)\left(8\pi^2 \frac{d}{dt}\log B^2\right) - \left(1, \frac{1}{2}\right)\left(8\pi^2 \frac{d}{dt}\log M_{\text{top}}^2\right) + \frac{1}{2}G_U, \tag{7.8}$$

The neutrino masses are obtained as

$$M_{\nu_e} = M_U a_N^{-1/2} a_U^{1/4}\left(\frac{B_Z}{B_U}\right)^{5/2}\left(\frac{M_U}{M_{\text{top}}}\right)^2 \simeq \lambda_{\nu_e}^{16} \simeq 3\times 10^{-9}\,\text{GeV}, \tag{7.9}$$

$$M_{\nu_\mu} = M_U a_N^{-1/2} a_U^{1/4}\left(\frac{B_Z}{B_U}\right)^{3/2}\left(\frac{M_U}{M_{\text{top}}}\right) \simeq \lambda_{\nu_\mu}^{9} \simeq 14\times 10^{-4}\,\text{GeV}, \tag{7.10}$$

$$M_{\nu_\tau} = M_U a_N^{-1/2} a_U^{1/4}\left(\frac{B_Z}{B_U}\right)\left(\frac{M_U}{M_{\text{top}}}\right)^{1/2} \simeq \lambda_{\nu_\tau}^{6} \simeq 13\times 10^{-2}\,\text{GeV} \tag{7.11}$$

This completes our calculation of the masses of the quarks and leptons in terms of known and calculable quantities η_F, satisfying the RG equations. The calculations given in these cases appear to be specific. It is absolutely necessary to write down the solutions of the RG equations for each quark and leptons separately, then alone, the contributions from the gauge interactions and the Yukawa mass coefficients A_{FH} can be ascertained.

However, the values of the coefficients η_F used above have been calculated, upto to the nearest fraction, from the cited rotational integers n_F in Table 6 and the top coupling coefficients N_F from the RG equations. They are given in the Table 7.

Table 7. Coefficients of η_F for fermions

Quarks	n_F	N_F	η_F	Leptons	n_F	N_F	η_F
c	3	3	0	e	8	0	30/24
u	7	3	1/2	μ	4	0	17/24
b	2	1	1/4	τ	3	0	11/24
s	4	0	3/4	ν_e	16	3	2
d	6	0	13/12	ν_μ	9	3	1
				ν_τ	6	3	1/2

8. Running masses of the fermions

The exact one loop solution of the RGE for change in mass values with energy is given by eq. (2.15). With $Z_F(t) = Y_F(t) - G_F(t) = 8\pi^2 (d/dt)\log B_F^2(t)$, this equation reduces to

$$\frac{M_{\text{top}}^2}{M_F^2(t)} = \frac{M_{\text{top}}^2}{M_U^2}\frac{B_F^2(t_X)}{B_F^2(t)} + \frac{A_F}{8\pi^2}\int_t^{t_X} dt_1 \frac{B_F^2(t_1)}{B_F^2(t)}, \tag{8 1}$$

and

$$M_F(t) = M_F(M_X)\frac{\dfrac{B_F(t)}{B_F(t_X)}}{\left(1+\dfrac{M_U^2}{M_{\text{top}}^2}\dfrac{A_F}{8\pi^2}\displaystyle\int_t^{t_X} dt_1 \frac{B_F^2(t_1)}{B_F^2(t_X)}\right)^{1/2}}. \tag{8 2}$$

If we take

$$B_F(t) = \lambda^{n_F(1-t/t_X)} = \exp\left(n_F\left(1-\frac{t}{t_X}\right)\log_e \lambda\right), \tag{8 3}$$

and

$$M_F(t) = M_U\lambda^{n_F}\left(1+\frac{M_U^2}{M_{\text{top}}^2}\frac{A_F}{8\pi^2}\int_t^{t_X} dt_1 \lambda^{2n_F(1-t_1/t_X)}\right)^{1/2}, \tag{8 4}$$

with

$$\int_t^{t_X} dt_1 \lambda^{-2n_F\frac{t_1}{t_X}} = \int_t^{t_X} dt_1 e^{-2n_F t_1/t_X \log_e \lambda}$$

$$= -\frac{t_X}{2n_F}\frac{1}{\log_e\lambda}\left[e^{-2n_F\log_e\lambda} - e^{-2n_F t/t_X \log_e\lambda}\right]$$

$$= -\frac{t_X}{2n_F}\frac{1}{\log_e\lambda}\left[\lambda^{-2n_F} - \lambda^{-2n_F t/t_X}\right], \tag{8 5}$$

we have, from eq. (8.2)

$$M_F(t) = M_F(M_X)\frac{\lambda^{n_F(1-t/t_X)}}{\left[1+\dfrac{M_F^2(M_X)}{M_{\text{top}}^2}\dfrac{A_F}{8\pi^2}\lambda^{2n_F}\lambda^{2n_F/t_X}\left(\lambda^{-2n_F}-\lambda^{-2n_F t/t_X}\right)\right]^{1/2}}. \tag{8.6}$$

We shall not use this for constructing the tables A simpler approximate form ignoring A_F terms of eq (8 6)

$$M_F(t) = M_F(M_X)\, \lambda^{n_F(1-t/33)} \tag{8 7}$$

is presented in tabular form in Tables 8, 9 and 10 This gives the same CKM matrix Now, we present our results in the following tables

Table 8. Variation of M_{top} (t) with $t - \log_e(\mu/M_Z)$

t	0	3	6	9	12	15	18	21	24	27	30	33
$M_{top}(t)$ (GeV)	175 88	166 98	159 60	153 25	147 61	142 46	137 64	133 02	128 51	124 51	119 56	115 00

Table 9. Running lepton masses in GeV t_X = 33

t/t_X	M_e	M_μ	M_τ	M_{ν_e}	M_{ν_μ}	M_{ν_τ}
0	0 00051	0 105	1 77	3×10^{-9}	$1\ 9\times10^{-4}$	0 018
0 1	0 00175	0 2115	2 687	$3\ 43\times10^{-8}$	$7\ 19\times10^{-4}$	0 043
0 2	0 006	0 426	4 079	$3\ 92\times10^{-7}$	$2\ 72\times10^{-3}$	0 104
0 3	0 0206	0 8575	6 192	$4\ 49\times10^{-6}$	$1\ 03\times10^{-2}$	0 25
0 4	0 2	1 726	9 399	$5\ 13\times10^{-5}$	$3\ 9\times10^{-2}$	0 6
0 5	0 242	3 476	14 267	$5\ 87\times10^{-4}$	0 1478	1 44
0 6	0 830	6 999	21 658	$6\ 72\times10^{-3}$	0 56	3 46
0 7	2 849	14 081	32 876	$7\ 68\times10^{-2}$	0 01	8 3
0 8	9 774	28 369	49 910	0 482	8 02	19 93
0 9	33 527	57 118	75 758	10 05	30 374	47 9
1 0	115 0	115 0	115 0	115 0	115 0	115 0

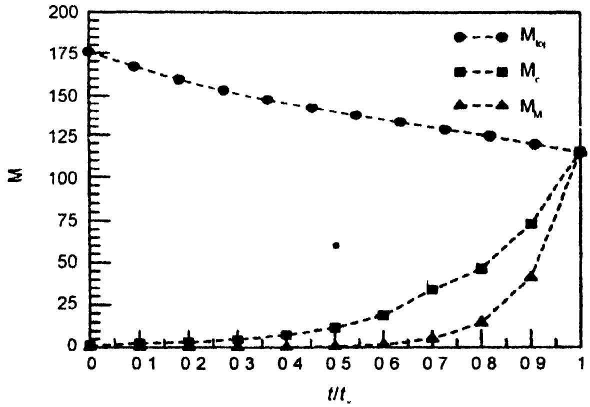

Figure 1. Variation of masses of up-quarks in GeV with t/t_X, $t = \log(\mu/M_Z)$ and t_X = 33

Table 10. Values of running quark masses in GeV (other than the top quark), t_X = 33

t/t_X	M_e	M_u	M_b	M_s	M_d
0 0	1 27	0 0042	4 23	0 159	0 0075
0 1	2 62	0 0116	5 88	0 307	0 0196
0 2	3 12	0 0324	8 18	0 593	0 051
0 3	4 9	0 09	11 39	1 146	0 135
0 4	7 7	0 25	15 85	2 213	0 354
0 5	12 0	0 695	22 05	4 276	0 9287
0 6	18 96	1 9307	30 687	8 26	2 434
0 7	34 16	5 3636	42 696	15 95	6 383
0 8	46 69	14 9	59 406	30 82	16 73
0 9	73 28	41 395	82 651	59 59	43 86
1 0	115 0	115 0	115 0	115 0	115 0

From the above values, we find that $\left(d^2M_{top}(t)\right)/dt^2$ is nearly zero The top mass variation is approximately

$$M_{top}(t) = \left| 175 - \frac{t}{33} \times 60 \right| \text{ GeV} \tag{8 8}$$

The graphs are much more revealing The variation of mass for the up quarks, including the top, have been shown in Figure 1, for the down in Figure 2, for the electrons in Figure 3 and for the neutrinos in Figure 4 All have been compressed in Figure 5 to allow a glance at the totality of the descent and ascent of the masses to the unification mass of

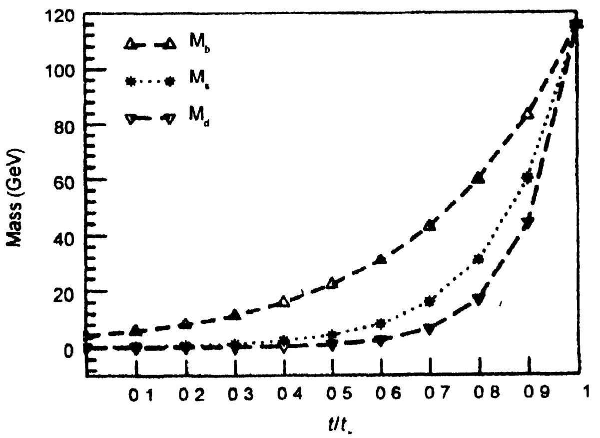

Figure 2. Variation of masses of down-quarks in GeV with t/t_X, $t = \log(\mu/M_Z)$ and t_X = 33

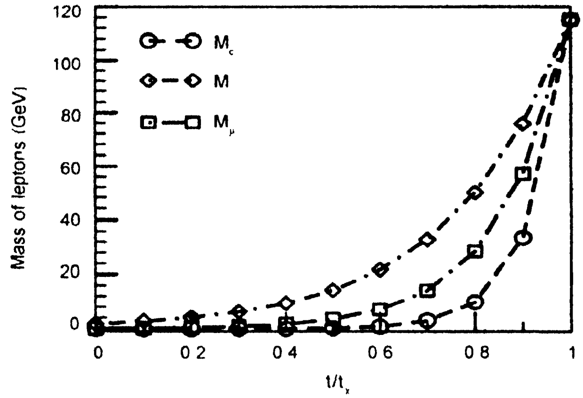

Figure 3 Variation of masses of electrons in GeV with t/t_X $t - \log(\mu/M_Z)$ and $t_X = 33$

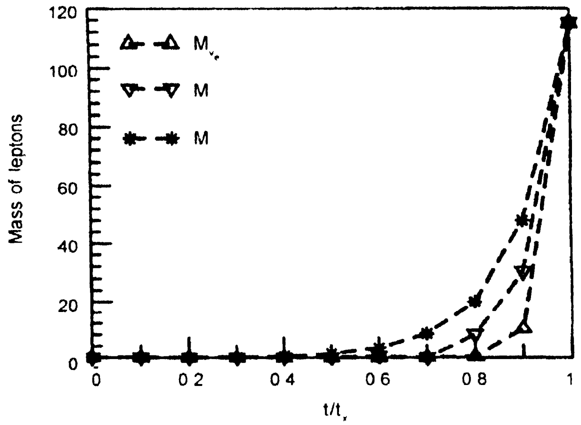

Figure 4 Variation of masses of neutrinos in GeV with t/t_X $t - \log(\mu/M_Z)$ and $t_X = 33$

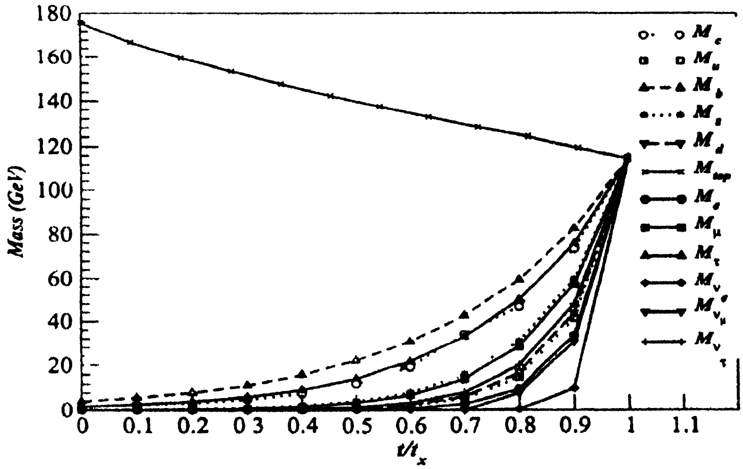

Figure 5. Variation of masses of all 12 fermions in GeV with t/t_X $t = \log(\mu/M_Z)$ and $t_X = 33$

115 GeV We believe that an exact analysis will not differ much from those presented in these Figures

9. Concluding remarks

Deo and Maharana [14] have already given near exact solutions for the one loop RGEs in MSSM They proved that all the fermions might have originated from a common mass $M_U \simeq 115$ GeV at the GUT energy of $M_X \simeq 2\,2 \times 10^{16}$ GeV in a perturbative scheme We have cited our work in the second of reference [14] We obtain $M_{\text{top}}(M_X)$ = 110 GeV If this is used for the bottom, we get the mass of the bottom to be 5 31 GeV These are examples of the two quark generations Gaillard and Lee, and other have remarked that lepton masses jump to quark mass values at high energy Thus these are evidences in support of a reverse look from the one original mass for all fermions It is interesting to note that the relations given in eq (1 20) are well satisfied in our approach

As the energy diminishes, the mass of the top increase to 175 GeV at the Z-meson mass M_Z, whereas, masses of all other quarks and tiny leptonic masses increase from their value at M_Z to M_U of each 115 GeV Hopefully, the exact nature of this variation could be obtained from RGE Hence by introducing an auxiliary function $B(t)$ for the nonlinear mixing terms of RGE, a very simplified expression has been obtained for both the mass values at $M_Z(t = 0)$ and their variation till M_X, while the masses of the fermions, which keep changing and attain the same final value at M_X

The success of the analysis of the Wolfenstein and the ratio parametrization of RRR given by eq (1 20) is clearly brought out by our solutions An extension to running masses at linearity level, leads to a very simple formula for the fermions other than the top,

$$M_F(t) \simeq M_U \left(\frac{M_F}{M_U} \right)^{\left(1-\frac{t}{t_x}\right)} = M_U\, \lambda^{n_f\left(1-\frac{t}{t_x}\right)}, \tag{9 1}$$

where $M_F^{\text{exp}} = M_F(M_Z) = \lambda^{n_F}$ The increase is exponential Much more exact analytic studies are needed to deduce the values of the n_F from RGE As a first step, we follow up the texture analysis procedure by introducing a similar, but not the same function $B(t)$ [13], which is such that $d \log B(t) = d \log M_{\text{top}}(t)$ The resulting non-uniqueness is taken advantage of introducing known gauge couplings and the values of n_F, the effect of gauge interaction have also been calculated

In the foregoing Sections 1 to 8, there has been two important omissions We have not remarked either about higher loop effects or about the threshhold corrections They may be very large due to the existence of a very heavy top Also there has been no mention of the CKM phases But, we work with three generations only So, in this case there is precisely, one phase angle ϕ This is defined from the Wolfenstein parametrisation as

$$\lambda = \left(\frac{M_d}{M_s} + \frac{M_u}{M_c} + 2\sqrt{\frac{M_d}{M_s}\frac{M_u}{M_c}} \cos\phi \right)^{\frac{1}{2}}, \tag{9 2}$$

and can be deduced by extending the Gatto-Sarton-Tonio-Oakes (GSTO) [18] to next leading order of solution of RGE, $\lambda = (M_d/M_s)^{1/2}$ Putting in the numbers n_F from Table 3 we get

$$\cos\phi \simeq -\frac{\lambda}{2} \simeq -0\,1, \qquad \phi \simeq 95^0 \tag{9 3}$$

This result is reasonable

It may be argued that there are too many parameters still in the guise of the rotation integers n_F even though λ could be determined Let us recall the case of the unification of the strong, the weak, the electromagnetic and the gravitational interactions One consciously omits gravity and lists the other three in order of their strengths Here the top mass, with the largest value, has been deduced from the knowledge of the unification mass and gauge couplings quite accurately For F = 2, the two successive rotations *i e* 0 to 4π gives n_F = 2 and $M_U\lambda^2$ falls near the mass of the bottom This process of complete successive rotations yields the masses of all the quarks and electrons except n_F = 5 However, there is a GUT prediction by Georgi and Jarlskog [3], that the ratio $M_s/M_d \simeq 25 \simeq \lambda^2$ This may be the reason why n_F jumps from 4 to 6 There is also another accurate prediction

$$\frac{M_d/M_s}{(1-M_d/M_s)^2} = 9\,\frac{M_e/M_\mu}{\left(1-M_e/M_\mu\right)^2} \tag{9 4}$$

With these GUT supplements, there is no other unknown additional parameter left The unification mass $M_U \simeq 115$ GeV is the only input Corroborating and in consonance with a lot of excellent work in neutrino physics have been done by many workers *e g* Babu Mohapatra and Barr [20], the neutrino masses fall with the rotational integers, determined for the neutrinos in the Table 5 in a nice way

The Figure 5 illustrates the success of our method to solve the RGEs The similarity of Figure 5 with gauge unification graph is striking The original ideas of Pati and Salam [1] is correct even with masses as well as the SUSY standard model We have suggested and calculated masses from one model when successfully implemented, SUSY standard model world can be characterised by only three parameters, the GUT mass $M_X \simeq 2\,2 \times 10^{16}$, the GUT coupling strength $\alpha_{\rm GUT} \simeq 1/24\,5$ and common-origin-fermion mass $M_U \simeq 115$ GeV This is a simplification beyond expectation Above the Planck energy, it is assumed that there was no matter, but only radiation At GUT energy, all the four interactions were presumably of the same strength In fact, $SU_L(2)$ gauge coupling strength is nearly the same in the range 10^2 to 10^{16} GeV, it is therefore natural to expect equal mass for all fermions at creation The possibility that all fermions, at one time, had one mass at about the GUT scale can not be ruled out

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